Cycloids in Louis I. Kahn’s Kimbell Art Museum at Fort Worth, Texas

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Does your hometown have any mathematical tourist attractions such as statues, plaques, graves, the café where the famous conjecture was made, the desk where the famous initials are scratched, birthplaces, houses, or memorials? Have you encountered a mathematical sight on your travels? If so, we invite you to submit to this column a picture, a description of its mathematical significance, and either a map or directions so that others may follow in your tracks.

Figure 1. Exterior views on the Kimbell Art Museum.
aries is one of the essential components of his architectural approach. Kahn also sought to optimize natural light for the occupants of the building.

Born on the island of Saaremaa, Estonia, in 1901, Kahn emigrated to the United States and practiced there until his death in 1974. Among his best-known buildings is the Kimbell Art Museum in Fort Worth, Texas (1966–1972), one of the most-admired architectural pieces of the time. Other masterpieces are the Salk Institute in California (USA), the Indian Institute of Management in Ahmedabad (India), and the National Assembly at Dacca (Bangladesh).

The Kimbell museum sits on the gentle slope of the Amon Carter Square Park, about two miles from downtown Fort Worth. The structure, covering a gross area of 120,000 square feet, lies in a trapezoid-shaped area of 9.5 acres, bounded by Camp Bowie Blvd., Arch Adams St., and West Lancaster Ave. There are a number of cultural institutions in the vicinity: the Amon Carter Museum of Western Art, the Fort Worth Museum of Science and History, and, directly opposite the Kimbell, along Arch Adams St., the new Modern Art Museum of Fort Worth.

The Kimbell Art Museum is home to one of the world’s finest art collections, with acquisitions ranging from antiquity to the twentieth century [Loud, 1987]. Among the interesting architectural features of the building are the unique vault roofs, which exhibit a rare example of a strategic use of the mathematical properties of the cycloid [Gast, 1998; Benedikt, 1991]. An art museum with a mathematical property: could a mathematical tourist ask for more?

A Man, a Plan: Kahn

Kahn started the design for the Kimbell in 1967, arriving at his final scheme around September 1968. August Komendant, Kahn’s structural engineer, describes this period as “very difficult and agonizing.” The design process seemed to have been an ordeal for Kahn: the first scheme was a square plan, with a circulatory street, an arcade around the complex, and polygonal ceilings. The second plan was an H-shape, with two connected structures covered by a walkway and curved roofs. The final layout was a C-type plan, with cycloid vaults, and that was the design that was executed.

The imposing structure opened four years later, in October 4, 1972. The museum incorporates the slope of the site and has entrances at both the lower and upper levels. Public access for pedestrians is offered on the west side, at the upper level, whereas the parking lots are located on the east side, at the lower level (accessed from Arch Adams Street). There is a central axis passing through the front and the rear entrances, the lobby and the stairs; two water pools outside the porticos are symmetric too, along that axis. This axial approach is typical of Kahn’s designs. On June 25, 1969, he wrote, in a letter to Mrs. Kimbell [Johnson 1975]:

Two open porticos flank the entrance court of terrace. In front of each portico is a reflecting pool which drops its water in a continuous sheet about 70 feet long in a basic two feet below. The sound would be gentle. The stepped entrance court passes between the porticos and their pools with a fountain around which one sits, on axis designed to be the source of the portico pools. The west lawn gives the building perspective.

Kahn continues:

The south garden is at a level 10 feet below the garden entrance approached by gradual stepped lawns sloped to be a place to sit to watch the performance of a play music or dance, the building with its arched silhouette acting as the backdrop of a stage. When not so in use it will seem only as a garden where sculpture acquired from time to time would be. The north garden, though mostly utilitarian, is designed with ample trees to shield and balance the south and north sides of the building.

The series of 16 structural units with their vaulted roof are supported inde-
Independently and clustered with preservation of their independent structural character. At the same time, each vault is an inseparable element of the whole, making the order in the composition clear: the Kahn approach.

These structural elements are grouped on a tripartite scheme, with northern, central, and southern parts. Whereas each of the north and the south parts has six vaults, the central part between them has four vaults. The permanent collection, an auditorium, a café and the galleries occupy the northern part. The central part, with only four vaults, provides an entrance court in which even the trees are arranged in an orderly fashion. It includes the main entrance hall, the lobby, a bookstore, the museum shop, and the library. The southern part holds galleries for temporary exhibitions, the reception hall, and a kitchen. All are arranged around a large open block dedicated to staff facilities including offices, shops, storage, conservation and photography studios, and workplaces for shipping and receiving. The three courts are all open to the sky and allow natural light into the museum. The largest square court on the northern side borders the café, while two smaller square courts are located on the south: one is two stories high so that the light penetrates to the floor below, to the conservator’s studio. Kahn provides an account of his intent for having separate gardens and naming each court separately [Latour 1991]:

Rather than using the equations of a cycloid, the design was made using a simple drawing of the curve generated by a point on a rolling circle, as shown in Figure 6. The rhythmic sequential cycloid structure is easily recognized from a distance. Each vault consists of 100 × 23 foot shells poured in place by post-tensioning cables and reinforced concrete, and sheathed by a calcium-lead layer. The roofs are about six feet apart, separated by flat-roofed channels for air conditioning and electrical distribution ducts. The height of a vault from the upper level is 20 feet and the height from the lower level is 40 feet. Each concrete cycloid vault is supported by four 2 × 2 foot corner concrete piers, thus providing column free spaces (Figure 7).

Seeing Cycloid Light

Each vault has a longitudinal 2 1/2 foot narrow slit at the roof, over the entire length of the structure, covered by acrylic Plexiglas. Perforated metal reflectors are installed under the light slit of the cycloid vault. Kahn’s series of preliminary sketches show an evolution, in which he experimented with various relationships between roof shapes and light reflectors (see figure

The cycloid vaults and the lighting reflectors were carefully designed to control the quality and intensity of daylight in the museum. Kahn writes [Johnson, 1975]:

The scheme of enclosure of the museum is a succession of cycloid vaults each of a single span [100] feet long and [23] feet wide, each forming the rooms with a narrow slit to the sky, with a mirrored shape to spread natural light on the side of the vault. This light will give a glow of silver to the room without touching the objects directly, yet give the comforting feeling of knowing the time of day.

The natural light was an integral part of the program for the museum design from the beginning [Loud 1987]. The Kimbell's first director, Richard Brown, was especially concerned that “natural light should play a vital part in illumination.” It may be conventional knowledge that natural light distracts the visual experience of viewers in a museum, but Kahn succeeded in using natural light as the main source for lighting the museum galleries. Natural light penetrates through the slit in the vault and reflects off the mirrored shape of the convex perforated metal reflectors back to the side of the interior vault and then diffuses to the exhibition room. Kahn describes 'the luminosity of solver' of the perforated metal reflectors with cycloid vaults as follows: . . . rather a new way of calling something; it is rather a new word entirely. It is actually a modifier of the light, sufficiently so that the injurious effects of the light are controlled to whatever degree of control is now possible. [Johnson 1975].

**Unusual Computational Complications**

The key to the museum design is the structural novelty of the cycloid roof, with a span of 100 ft, supported by four corner columns. The monolithic structure is only four inches thick; so thin it looked “fragile,” even to Kahn himself. The roof, with its longitudinal 2½ foot narrow slit at the apex of each vault, is braced by cross concrete struts every 10 feet, transferring the load along the vault, longitudinally and vertically, to the lower corner of the vault and to the four corner columns. As shown in Figure 11, the forces of the longitudinal load are absorbed in two ways by three post-tensioning cables, a technique commonly employed to improve the load-bearing properties of concrete by incorporating high-tensile steel stretched cables into the structure. The diaphragms at the end absorb the forces of the vertical load.

Traditionally, a barrel roof can be supported along the longitudinal edges or on the curved ends. When a barrel is supported along the longitudinal edges, it behaves like a row of arches, one next to the other, developing horizontal forces pushing outward and absorbed by buttress walls. When a barrel vault is supported on its curved ends, it behaves like a beam conveying the load to the walls at the end and to the ground (Salvadori, 1980). However, in the Kimbell Art Museum design, the structure of the roof is pushed beyond the conventional knowledge of typical vault structures. Kahn’s design employs an unconventional mixture of techniques using those complicated post-tensioning cables and diaphragms. It seems Komendant’s contribution to the cycloid structure for the museum was essential (Komendant, 1975; Leslie, 2005).

Mathematicians have long admired the distinctive properties of the cycloid, and, more generally of the cycloid family, including epicycloids, hypocycloids, epitrochoids, hypotrochoids, or cardioids. The length of one arch of a cycloid is four times the diameter of the rolling circle, while the area under the arch is three times the area of the circle.
Figure 5. South courtyard with garden outdoor water fountain.

Figure 6. The generation of the cycloid (after Louis I. Kahn).
Figure 7. Section and elevation of the museum (left) and a joint where roof and corner column meet (right).

Figure 8. Detailed photo showing the roof structure of the museum (left), and porticos with a side access path and reflecting pool (right).

Figure 9. A series of Kahn's preliminary sketches for the roof, made in 1967, and how the light should infiltrate (Redrawn by the author).

Figure 10. A gallery, showing the transparent ceiling reflector (left), and a café and gallery areas, showing the north garden court with Maillol’s “L’Air” (right).
The cycloid shape of the Kimbell Art Museum is perhaps the most singular example ever applied in building design. The visitor feels the full effect of a quintessentially dignified cycloid space form embodying a mathematical curve into a physical building. Kahn used different types of arcs such as circle, semi-circle, and circle sectors in other buildings, but he repeated his cycloid roof design and the natural light fixture of the museum in only one other building, the Wolfson Center for Mechanical and Transportation Engineering (1968–1977) on the university campus of Tel Aviv.

ACKNOWLEDGMENT
We thank Kazi Ashiraf for allowing the use of his Kimbell Museum photographs in this article.

REFERENCES
The Kimbell Art Museum website: http://www.kimbellart.org/
AU1
This is difficult to resize. Perhaps figure 4 could be labeled to show some of the elements?

AU2
Is this the blue, yellow, or green court or something else?